Application of Dijkstra's Algorithm to find the Shortest Path of Road Map

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Abstract

In human life, people travel to everywhere about work, visit, study and economy. So, they choose to reach with minimum distance by the shortest path. This paper was aimed to find the shortest path that the people were reached quickly from a starting place to the target place in the sparse graph with weighted edges by using Dijkstra's algorithm. The calculated results of the shortest path from Yadanabon bridge(Y) to Kaunghmudaw pagoda (P) in Sagaing and the shortest path from Sagaing 700 Anniversary Football Pitch(S) to Maha Sigon Gyi Pagoda (M) in Parami quarter, Sagaing were 10.5 km and 950 m, respectively. Dijkstra's algorithm was very useful method to find the shortest paths of traffic information systems, road maps, network and so on. This method runs faster than Floyd's algorithm to solve the problems for the sparse graphs with weighted edges.

Keywords: Shortest path, Dijkstra's algorithm, vertices, edges, distance, sparse graph

1. Introduction

People are travelling in different ways such as by car, train, plane and so on from one country to another in the world. There are many villages and cities in Myanmar. Thus, People was connected with many paths from one city to another in Myanmar. The aims of this research was to find the shortest path how to reach quickly the people from a starting place to another and to illustrate the shortest path by using Dijkstra's algorithm. There are many methods to calculate the shortest path in mathematics. In this research, the shortest path was applied by Dijkstra's algorithm. Thus the two experiments were showed the sparse graphs including the distances and calculated by Dijkstra's algorithm. If the people solved the problems with this method, they get the correct shortest path. When they observe the method finding the shortest path, they apply to the "road map", "networks" in electrical engineering, "structures" in civil engineering, "telecommunication networks", "orgnanizational structures" in economics, and so on by using Dijkstra's algorithm.

2. Related Works

Algorithm is a part of road maps and it also show people can travel from one place to another. Dijkstra's algorithm propose in a map based on a new data structure described with the relationships among the different places represented a community. This paper was studied the two sparse graphs including distances and spaces by using Dijkstra's algorithm. Nowadays, complexity road map and improved traffic can find out the best path ways from one location to another. The problems can calculated by many search algorithms and Dijkstra's algorithm, Floyd's algorithm know well. Both Dijkstra's algorithm and Floyd's algorithm may be used for finding the shortest path between vertices. The biggest difference is that Floyd's algorithm finds the shortest path between all vertices and Dijkstra's algorithm finds the shortest path between a single vertex and all other vertices. Dijkstra's algorithm more useful to find the shortest path for sparse graphs with weighted edges because it runs faster than Floyd's algorithm. For other graphs it is better to use Floyd's algorithm to compute the shortest path.[2]

3. Definition of Graph

A graph G consists of points was called vertices (V), and lines connecting them, were called edges (E), such that each edge connects two vertices, called the endpoints of the edge.
We write G = (V, E)
Excluded are isolated vertices (vertices that are not endpoints of any edge), loops (edges whose endpoints coincide), and multiple edges (edges that have both endpoints in common). [4]

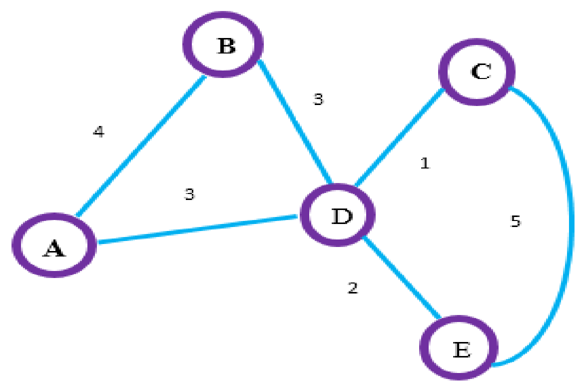


Figure 1. Weighted graph consisting of 5 vertices and 6 edges with distance

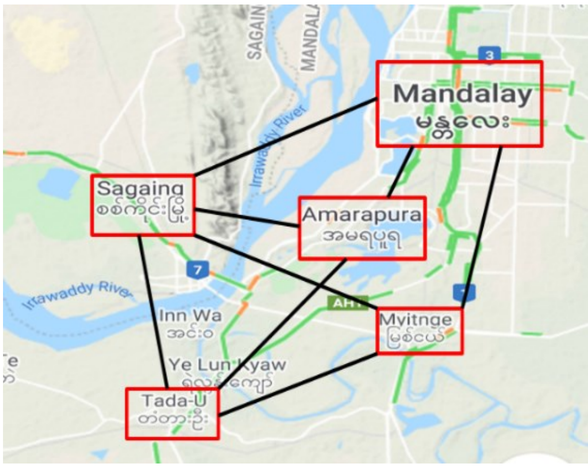


Figure 2. Unweighted graph connected with five cities.

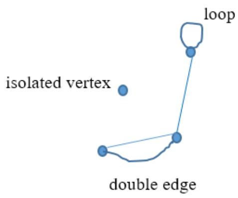


Figure 3. Isolated vertex, double edge, loop (excluded by definition)

3.1. Digraph (Directed Graph)

A digraph G = (V, E) is a graph in which each edge e = (i ,j) has a direction from its “initial point” i to its “terminal point” j .[4]

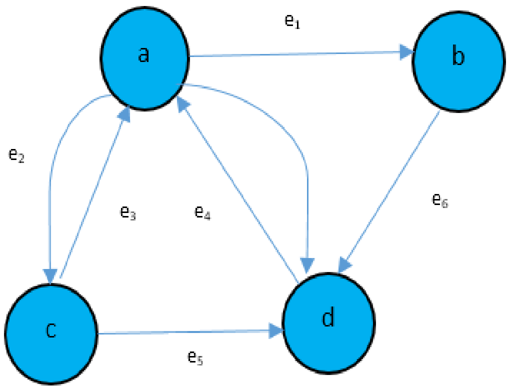


Figure 4. Digraph connected with four points

3.2. Sparse and Dense Graphs

A sparse graph has relatively few edges between nodes. A dense graph has relatively many edges between nodes.[6]

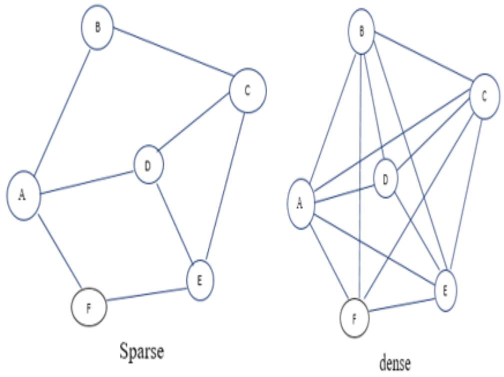


Figure 5. Sparse and dense graphs connected with six points

3.3. Shortest Path Problem

The problem of finding a path that the minimum distance (weight) between points (spaces or cities) on roads map is called the shortest path problem.

4. Dijkstra’s Algorithm

Dijkstra’s algorithm is mainly used to find the shortest path from a starting place to the target place in a weighted graph. When Dijkstra’s algorithm is applied, it creates a tree of shortest path from a starting place source to all the other places in the graph. It can be either a directed or undirected graph. A few constraints for application of Dijkstra’s algorithm, the graph must be a weighted graph with non-negative weighted edges. Dijkstra’s algorithm do not work for negative edges. [5]

Dijkstra’s algorithm is algorithm used to determine shortest path from a to z in a graph. Graph (G) is a simple graph connected with all distance positive. G has vertices a =…=z and edges (i,j) having lengths ℓ_{ij} where $\ell_{ij} = \infty$ if {i,j}is not an edge in G. Input are number of vertices(a to z) ,edges (i,j) and lengths ℓ_{ij} and output are lengths L_j of the shortest paths $a \to j$,(j =b,…,z).In calculating , step 1 is initial step with $L_a = 0$ and $\widetilde{L}_j = \ell_{aj}$ ($=\infty$ if there is no edge (a, j) in G) where $j(= b, c, ..., z)$.The results are visited $= \{a\}$ and unvisited $= \{b, c, ..., z\}$.Next, the step 2 is a permanent label with $L_k = \min\{\widetilde{L}_b, \widetilde{L}_c, ..., \widetilde{L}_z\}$. It will get k value. Delete k from unvisited and include it in visited. So visited= $\{a, k\}$, unvisited= $\{b, ..., k-1, k+1, ..., z\}$. If unvisited= ϕ (empty), then, output stop. If unvisited is not empty, this problem calculated the step 3. It is update temporary labels with $\widetilde{L}_j = \min\{\widetilde{L}_j, L_k + \ell_{kj}\}, \forall j$, in unvisited (that is take the smaller of $\widetilde{L}_j$ and $L_k + \ell_{kj}$ as your new $\widetilde{L}_j$). Next, it calculated by step 2.This problem was calculated the result step by step with this method. [4]

5. Experiment-1

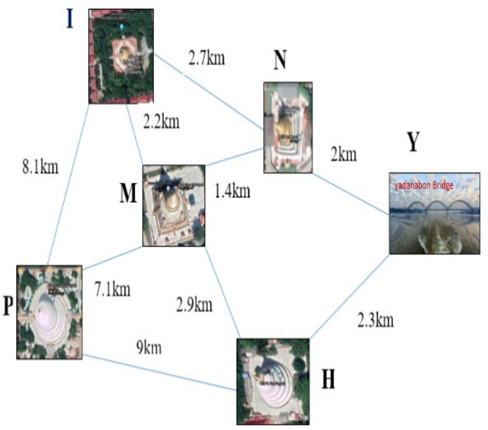


Figure 6. Road map with P, N, M, H, I, Y vertices

where, P=Kaunghmudaw Pagoda
N=Ma Shi Khana Pagoda
M=Maha Sigon Gyi Pagoda
H=Htu Pa Yon Pagoda
I=International Buddhish Academy
Y=Yadanabon Bridge

In this graph, vertices were P, N, M, H, I, Y and Edges were distances from one vertex to another.

This experiment-1 was determined the shortest path from Yadanabon bridge(Y) to Kaunghmudaw Pagoda(P) in figure 6.

Result,
I list the steps and computation.

1. $\widetilde{L}_Y = 0, \widetilde{L}_N = 2, \widetilde{L}_H = 2.3, \widetilde{L}_I = \widetilde{L}_M = \widetilde{L}_P = \infty$

Visited={Y}, Unvisited= {N,H,I,M,P}

2. $L_N = \min\{\widetilde{L}_N, \widetilde{L}_H, \widetilde{L}_I, \widetilde{L}_M, \widetilde{L}_P\} = \min\{2, 2.3, \infty, \infty, \infty\} = 2$ k=N, Visited={Y,N} , Unvisited={ H,I,M,P}

3. $\widetilde{L}_H = \min\{\widetilde{L}_H, L_N + \ell_{NH}\} = \min\{2.3, 2+\infty\} = 2.3$

$\widetilde{L}_I = \min\{\widetilde{L}_I, L_N + \ell_{NI}\} = \min\{\infty, 2+2.7\} = 4.7$

$\widetilde{L}_M = \min\{\widetilde{L}_M, L_N + \ell_{NM}\} = \min\{\infty, 2+1.4\} = 3.4$

$\widetilde{L}_P = \infty$

2. $L_H = \min\{\widetilde{L}_H, \widetilde{L}_I, \widetilde{L}_M, \widetilde{L}_P\} = \min\{2.3, 4.7, 3.4, \infty\} = 2.3$

k=H, Visited={Y,N,H} , Unvisited={ I,M,P}

3. $\widetilde{L}_M = \min\{\widetilde{L}_M, L_H + \ell_{HM}\} = \min\{3.4, 2.3+2.9\} = 3.4$

$\widetilde{L}_P = \min\{\widetilde{L}_P, L_H + \ell_{HP}\} = \min\{\infty, 2.3+9\} = 11.3$

$\widetilde{L}_I = 4.7$

2.
$L_M = \min\{\widetilde{L}_I, \widetilde{L}_M, \widetilde{L}_P\} = \min\{4.7, 3.4, 11.3\} = 3.4$

k=M, Visited={Y,N,H,M} , Unvisited={ I,P}

3. $\widetilde{L}_I = \min\{\widetilde{L}_I, L_M + \ell_{MI}\} = \min\{4.7, 3.4+2.2\} = 4.7$

$\widetilde{L}_P = \min\{\widetilde{L}_P, L_M + \ell_{MP}\} = \min\{11.3, 3.4+7.1\} = 10.5$ 2.

$L_I = \min\{\widetilde{L}_I, \widetilde{L}_P\} = \min\{4.7, 10.5\} = 4.7$

k=I, Visited={Y,N,H,M,I} , Unvisited={P}

3.
$\widetilde{L}_P = \min\{\widetilde{L}_P, L_I + \ell_{IP}\} = \min\{10.5, 4.7+8.1\} = 10.5$ 2.

$L_P = 10.5$

k=P, Visited={Y,N,H,M,I,P} , Unvisited=∅

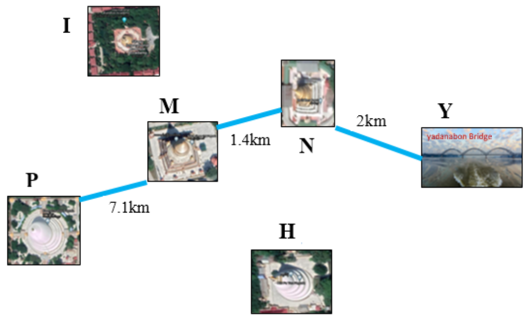


Figure 7. The shortest path (blue colour) from Yadanabon Bridge (Y) to Kaunghmudaw Pagoda (P)

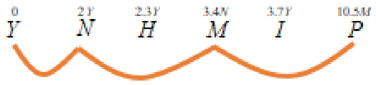
So, the distance of the shortest path from Y to P: Y→N →M →P = 10.5 km.

(or)

Table 1. The shortest path by Dijkstra's algorithm for experiment-1

No	k	Visited	$\widetilde{L}_j = \min\{\widetilde{L}_j, L_k + \ell_{kj}\}$	Unvisited
0				$\overset{0}{Y}\ \overset{\infty}{N}\ \overset{\infty}{H}$ $\overset{\infty}{I}$ $\overset{\infty}{M}\ \overset{\infty}{P}$
1	Y	$\overset{0}{Y}$	$\widetilde{L}_N = \min\{\infty, 0+2\} = 2$ $\widetilde{L}_H = \min\{\infty, 0+2.3\} = 2.3$	$\overset{2Y}{N}\ \overset{2.3Y}{H}$ $\overset{\infty}{I}\ \overset{\infty}{M}\ \overset{\infty}{P}$
2	N	$\overset{0}{Y}\ \overset{2Y}{N}$	$\widetilde{L}_I = \min\{\infty, 2+2.7\} = 4.7$ $\widetilde{L}_M = \min\{\infty, 2+1.4\} = 3.4$	$\overset{2.3Y}{H}\ \overset{4.7N}{I}$ $\overset{3.4N}{M}\ \overset{\infty}{P}$
3	H	$\overset{0}{Y}\ \overset{2Y}{N}\ \overset{2.3Y}{H}$	$\widetilde{L}_M = \min\{3.4, 2.3+2.9\} = 3.4$ $\widetilde{L}_P = \min\{\infty, 2.3+9\} = 11.3$	$\overset{4.7N}{I}\ \overset{3.4N}{M}$ $\overset{11.3H}{P}$
4	M	$\overset{0}{Y}\ \overset{2Y}{N}\ \overset{2.3Y}{H}$ $\overset{3.4N}{M}$	$\widetilde{L}_I = \min\{4.7, 3.4+2.2\} = 4.7$ $\widetilde{L}_P = \min\{11.3, 3.4+7.1\}$ $= 10.5$	$\overset{4.7N}{I}$ $\overset{10.5M}{P}$
5	I	$\overset{0}{Y}\ \overset{2Y}{N}\ \overset{2.3Y}{H}$ $\overset{3.4N}{M}\ \overset{4.7N}{I}$	$\widetilde{L}_P = \min\{10.5, 4.7+8.1\}$ $= 10.5$	$\overset{10.5M}{P}$
6	P	$\overset{0}{Y}\ \overset{2Y}{N}\ \overset{2.3Y}{H}$ $\overset{3.4N}{M}\ \overset{4.7N}{I}$		-

		$\overset{10.5M}{P}$		



If Dijkstra's algorithm was calculated with table, it can be solved the result more easily and quickly.
Therefore,

The distance of the shortest path from Y to P is 10.5 km(Y→N→M→P).

6. Experiment-2

Figure 8. Map included sagaing 700 anniversary football pitch and maha sigon gyi pagoda in google map

This experiment-2 was determined a shortest path between Sagaing 700 Anniversary Football Pitch and Maha Sigon Gyi Pagoda in Parami quarter, Sagaing.

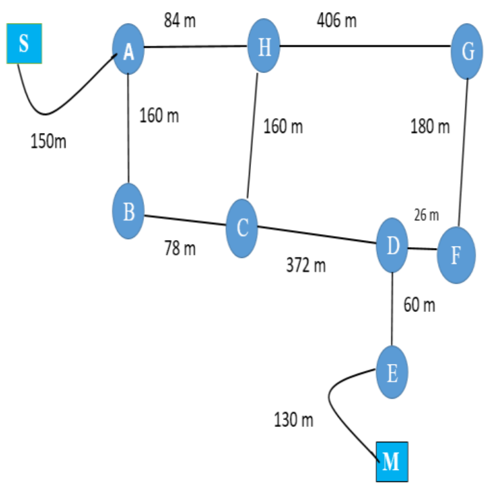


Figure 9. Distances of the roads between S and M in Parami quarter, Sagaing.

where, S = Sagaing 700 Anniversary Football Pitch
M = Maha Sigon Gyi Pagoda

Road's vertices = A,B,C,D,E,F,G,H.
Result,

Table 2. The shortest path by Dijkstra's algorithm for experiment-2

No	k	Visited	Unvisited
0			$\overset{0}{S}\ \overset{\infty}{A}\ \overset{\infty}{B}\ \overset{\infty}{C}\ \overset{\infty}{D}\ \overset{\infty}{E}\ \overset{\infty}{F}\ \overset{\infty}{G}\ \overset{\infty}{H}\ \overset{\infty}{M}$
1	S	$\overset{0}{S}$	$\overset{150S}{A}\ \overset{\infty}{B}\ \overset{\infty}{C}\ \overset{\infty}{D}\ \overset{\infty}{E}\ \overset{\infty}{F}\ \overset{\infty}{G}\ \overset{\infty}{H}\ \overset{\infty}{M}$
2	A	$\overset{0}{S}\ \overset{150S}{A}$	$\overset{310A}{B}\ \overset{\infty}{C}\ \overset{\infty}{D}\ \overset{\infty}{E}\ \overset{\infty}{F}\ \overset{\infty}{G}\ \overset{234A}{H}\ \overset{\infty}{M}$
3	H	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}$	$\overset{310A}{B}\ \overset{394H}{C}\ \overset{\infty}{D}\ \overset{\infty}{E}\ \overset{\infty}{F}\ \overset{640H}{G}\ \overset{\infty}{M}$
4	B	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}\ \overset{310A}{B}$	$\overset{388B}{C}\ \overset{\infty}{D}\ \overset{\infty}{E}\ \overset{\infty}{F}\ \overset{640H}{G}\ \overset{\infty}{M}$
5	C	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}\ \overset{310A}{B}\ \overset{388B}{C}$	$\overset{760C}{D}\ \overset{\infty}{E}\ \overset{\infty}{F}\ \overset{640H}{G}\ \overset{\infty}{M}$
6	G	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}\ \overset{310A}{B}\ \overset{388B}{C}\ \overset{640H}{G}$	$\overset{760C}{D}\ \overset{\infty}{E}\ \overset{820G}{F}\ \overset{\infty}{M}$
7	D	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}\ \overset{310A}{B}\ \overset{388B}{C}\ \overset{640H}{G}\ \overset{760C}{D}$	$\overset{820D}{E}\ \overset{786D}{F}\ \overset{\infty}{M}$
8	F	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}\ \overset{310A}{B}\ \overset{388B}{C}\ \overset{640H}{G}\ \overset{760C}{D}\ \overset{786D}{F}$	$\overset{820D}{E}\ \overset{\infty}{M}$
9	E	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}\ \overset{310A}{B}\ \overset{388B}{C}\ \overset{640H}{G}\ \overset{760C}{D}\ \overset{786D}{F}\ \overset{820D}{E}$	$\overset{950E}{M}$
10	M	$\overset{0}{S}\ \overset{150S}{A}\ \overset{234A}{H}\ \overset{310A}{B}\ \overset{388B}{C}\ \overset{640H}{G}\ \overset{760C}{D}\ \overset{786D}{F}\ \overset{820D}{E}\ \overset{950E}{M}$	-

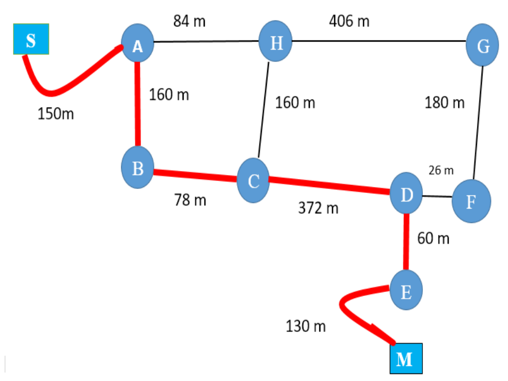


Figure 10. Shortest path (red colour) from S to M in the distanced graph.

The distance of the shortest path from S to M was 950 m (S→A→B→C→D→E→M).

7. Conclusion

In this research, Dijkstra's algorithm was used to find the shortest path in many paths to go from a starting place to another. If people go from Yadanabon bridge to Kaunghmudaw Pagoda, it showed in figure(7), the shortest path was 10.5 km and the shortest path connected from Sagaing 700 Anniversary Football Pitch to Maha Sigon Gyi Pagoda in figure(10) was 950m. Thus they can calculate quickly the shortest path by using Dijkstra's algorithm. In addition, Dijkstra's algorithm was used to find the shortest path in the "road map", "network", "flight Agenda" and so on. Therefore it was useful method to find the shortest path of the communications network in human's life.

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