IMAGE COMPRESSION USING HAAR WAVELET TRANSFORM

Mg Thae Mg, Daw Thu Zar Aung
Computer University (Myitkyina)
blue.guy1984@gmail.com

ABSTRACT

Some digital images need a large amount of memory space, so that precious memory in computer is waste. Image compression is an essential technique for storage and network transmission. This paper proposes implementation of image compression using Haar wavelet transform. The input image is acquired by some digital imaging device such as a digital camera, a scanner or imaging sensor. If the input image is color image (RGB image), it is need to change as grayscale image. The resultant image is transformed by using Haar wavelet transform method. Select non-negative threshold value (ϵ). The resultant transformed image is compared with non-negative threshold value (ϵ). When the values of transformed image are less than threshold value (ϵ), it is need to replace zero in the value of transformed image.

1. INTRODUCTION

Image compression is used to minimize the amount of memory needed to represent an image. Images often require a large number of bits to represent them, and if the image needs to be transmitted or stored, it is impractical to do so without somehow reducing the number of bits. Three techniques of image compression are pixel coding, predictive coding, and transform coding. The idea behind pixel coding is to encode each pixel independently.[4] The pixel values that occur more frequently are assigned shorter code words (fewer bits), and those pixel values that are more rare are assigned longer code words. This makes the average code word length decrease. The image compression technique most often used is transform coding. A typical image's energy often varies significantly throughout the image, which makes compressing it in the spatial domain difficult; however, images tend to have a compact representation in the frequency domain packed around the low frequencies, which makes compression in the frequency domain more efficient and effective. Transform coding is an image compression technique that first switches to the frequency domain, then does it's compressing. The transform coefficients should be decorrelated, to reduce redundancy and to have a maximum amount of information stored in the smallest space. These coefficients are then coded as accurately as possible to not lose information. Haar wavelet transform allows information to be encoded according to "level of detail". Haar wavelet transform works averaging and differencing method.[1] A grayscale image that is 256 X 256 pixels has 65536 elements to store, and 640 X 480 color image has nearly a million. The size of these files can also make downloading from the internet a lengthy process. Image compression plays a vital role in several important and diverse applications, including televideoconferencing remote sensing, medical imaging and magnetic resonance imaging and many more. The wavelet transform, developed for signal and image processing, has been extended for use on relational efficient, exactly reversible without the edge effects, it is fast and simple. The Haar transform is widely used in these days in wavelet analysis.

2. HAAR WAVELET

In mathematics, the Haar wavelet is a certain sequence of functions. It is now recognized as the first known wavelet. This sequence was proposed in 1909 by Alfred Haar. Haar used these functions to give an example of a countable orthonormal system for the space of square-integrable functions on the real line. The study of wavelets, and even

the term "wavelet", did not come until much later. As a special case of the Daubechies wavelet, it is also known as **D2**. The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. [7]This property can, however, be an advantage for the analysis of signals with sudden transitions, such as monitoring of tool failure in machines.
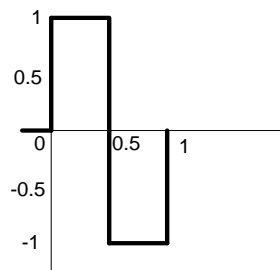
Figure 1. Haar wavelet

The Haar wavelet's mother wavelet function $\Psi(t)$ can be described as

$$\Psi(t)= \quad 1 \quad \text{if} \quad 0 \leq t < 1/2$$
$$\Psi(t)= \quad -1 \quad \text{if} \quad 1/2 \leq t < 1$$
$$\Psi(t)= \quad 0 \quad \text{if} \quad \text{otherwise}$$

and its scaling function $\varphi(t)$ can be described as

$$\phi(t)= \quad 1 \quad \text{if} \quad 0 \leq t < 1$$
$$\phi(t)= \quad 0 \quad \text{if} \quad \text{otherwise}$$

3. PROPERTIES OF HAAR WAVELET

The Haar wavelet has several notable properties:

1. Any continuous real function can be approximated by linear combinations of $\phi(t), \phi(2t), \phi(4t), \dots, \phi(2^{\kappa}t)$, and their shifted function. This extends to those functions spaces where any function therein can be approximated by continuous function.

2. Any continuous real function can be approximated by linear combinations of the constant function, $\Psi(t), \Psi(2t), \Psi(4t), \dots, \Psi(2^{k}t)$ …. and their shift functions.

3. Orthogonality in the form

$$\int_{-\infty}^{\infty} 2^m \psi(2^m t - n)\psi(2^{m1} t - n_1)dt = \delta_{m,m1}\delta_{n,n1}$$

$\delta_{i,j}$ represents the Kronecker delta. The dual function of $\psi(t)$ is $\psi(t)$ itself.

4. Wavelet/scaling functions with different scale m have a functional relationship:

$$\varphi(t) = \varphi(2t) + \varphi(2t - 1)$$
$$\psi(t) = \varphi(2t) - \varphi(2t - 1)$$

5. Coefficients of scale m can be calculated by coefficients of scale m+1:

If
$$\chi_w(n,m) = 2^{m/2} \int_{-\infty}^{\infty} x(t)\phi(2^m t - n)dt$$

and $X_w(n,m) = 2^{m/2} \int_{-\infty}^{\infty} x(t)\psi(2^m t - n)dt$ then

$$\chi_w(n,m) = \sqrt{\frac{1}{2}}(\chi_w(2n,m+1) + \chi_w(2n+1,m+1))$$

4. HAAR WAVELET TRANSFORM

4.1. Averaging and differencing method

The first method of transformation is called averaging and differencing method. [1], [2], [3], [5],[6]. Averaging and differencing is also called Haar wavelet transform. The averaging and differencing method is very effective for calculations. [1] By using this method, the calculations can quickly become quite tedious. [2] Let's we consider the following grayscale image.
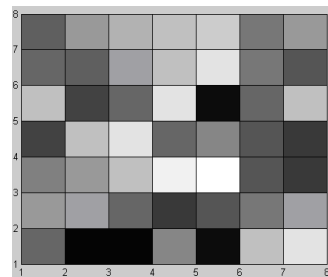
Figure 1. 8 x 8 grayscale image

The first step in image compression, 8 x 8 image is converted into matrix form using MATLAB or other mathematics program. 8 x 8 image matrix of grayscale image is represented in Figure 2.

$$I= \begin{bmatrix} 40 & 3 & 2 & 50 & 6 & 70 & 80 & 30 \\ 55 & 60 & 40 & 23 & 32 & 41 & 60 & 22 \\ 44 & 55 & 67 & 88 & 90 & 31 & 22 & 40 \\ 25 & 66 & 81 & 40 & 50 & 30 & 22 & 41 \\ 67 & 25 & 37 & 81 & 5 & 40 & 71 & 81 \\ 40 & 35 & 60 & 70 & 80 & 41 & 32 & 11 \\ 33 & 56 & 65 & 70 & 72 & 43 & 55 & 66 \\ 45 & 67 & 88 & 90 & 91 & 20 & 35 & 36 \end{bmatrix}$$

Figure 2. 8 x 8 image matrix

We extract the first row of I and named it I'. Therefore I'= [40 3 2 50 6 70 80 30]. The data string has length $8=2^3$, there will be three steps to the transform process. If the string were 2^n long, there would be n steps in the process. Four pairs of numbers will become. They are 40 and 3 , 2 and 50 , 6 and 70 , 80 and 30. To obtain the approximation coefficient of I_1', these pairs of numbers must be averaged. (40+3) / 2=21.5, (2+50) / 2=26, (6+70) / 2=38, (80+30) / 2=55. To obtain detail coefficient we subtract these averages (21.5, 26, 38 and 55) from the first member of each pair. 40-21.5=18.5, 2-26=-24, 6-38=-32, 80-55=25. The new data string I_1' become, I_1' = [21.5 26 38 55 18.5 -24 -32 25]. The detail coefficients are repeated in each subsequent transformation of this data string. In I_1' string, two pairs of numbers are 21.5 and 26, 38 and 55. To obtain the approximation coefficient of I_2', these pairs of numbers must be averaged. (21.5+26) / 2=23.75 and (38+55)/ 2 = 46.5. To obtain detail coefficient of I_2' we subtract these averages (23.75 and 46.5) from the first number of each pair. The new string I_2' is I_2'=[23.75 46.5 -2.25 -8.5 18.5 -24 -32 25]. The last four elements of I_2' are identical to I_1'. The last step our data string becomes: I_3'=[35.125 -11.375 -2.25 -8.5 18.5 -24 -32 25]. To obtain the first entry of I_3' we average two approximation

coefficients of I_2' (23.75 and 46.5). To obtain the second entry of I_3' we subtract the average value of I_2' from the first number of I_2'. The six detail coefficients of I_3' are followed by the detail coefficients of I_2'. I_1'=[21.5 26 38 55 18.5 -24 -32 25] I_2'=[23.75 46.5 -2.25 -8.5 18.5 -24 -32 25] I_3'=[35.12 -11.37 -2.25 -8.5 18.5 -24 -32 25] We treat each row of matrix I to obtain row transformed matrix is

$$\begin{bmatrix} 35.12 & -11.37 & -2.25 & 8.50 & 18.5 & -24 & -32 & 25 \\ 41.62 & -28.75 & 13 & 2.25 & -.25 & 8.5 & -4.5 & 19 \\ 54.62 & 8.875 & -14 & 14.75 & -5.5 & -10.5 & 29.5 & -9 \\ 44.37 & 8.63 & -7.5 & 4.25 & -20.5 & 20.5 & 10 & -9.5 \\ 50.87 & 1.62 & -6.5 & -26.75 & 21 & -22 & -17.5 & -5 \\ 46.12 & 5.12 & -13.75 & 19.5 & 2.5 & -5 & 19.5 & 10.5 \\ 57.5 & -1.5 & -11.5 & -1.5 & -11.5 & -2.5 & 14 & -5.5 \\ 59 & 13.5 & -16.5 & 10 & -11 & -1 & 35.5 & -.5 \end{bmatrix}$$

After treating each row of the matrix using averaging and differencing method, we also treat each column of the matrix using averaging and differencing method. The row and column transformed matrix (T) is

$$\begin{bmatrix} 141.3 & 53.68 & 81.71 & -35.7 & 154.3 & -2.18 & 16.6 & -118 \\ 64 & -13.4 & 21.87 & -15.6 & 5.1 & -9.46 & -28.1 & -1.3 \\ 50.5 & 15.2 & 46.4 & -28.2 & 66.1 & -20.4 & -4.1 & -12.8 \\ -109 & 15.3 & 10.3 & 17.7 & -17.9 & -40.3 & 31.8 & 45.2 \\ 113 & 18.7 & 32.2 & -46.5 & 72.1 & -8.5 & 2.23 & -96.7 \\ -49.4 & 0.21 & -9.4 & -2.07 & 20.9 & 8.49 & -2.77 & 30.2 \\ 28.38 & -14.3 & 5.4 & -7.87 & 48.3 & 18.8 & -14.1 & -25 \\ -180 & -41.4 & -8.4 & 18.35 & -39.3 & 1.22 & 24.5 & 85.5 \end{bmatrix}$$

4.2. Linear algebra method

In averaging and differencing method,
I'=[60 52 59 8 28 0 53 19 24].
I_1'=[56 33.5 40.5 21.5 4 25.5 -12.5 -2.5]
I_2'=[44.75 31 11.25 9.5 4 25.5 -12.5 -2.5]
I_3'=[37.875 6.875 11.25 9.5 4 25.5 -12.5 -2.5]
In linear algebra method, the transformation from I' to I_1' is illustrated in the following equation

3

$$I_1^{'} = I' A_1 \text{ , Where,} A_1 \text{ is}$$

$$A_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

The transformation from $I_1^{'}$ to $I_2^{'}$ is can be written as $I_2^{'} = I_1^{'} A_2$. Where A_2 is

$$A_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation from $I_2^{'}$ to $I_3^{'}$ is $I_3^{'} = I_2^{'} A_3$. Where A_3 is

$$A_3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By using transformation matrix W to transform I' to $I_3^{'}$. Where $W = A_1 A_2 A_3$.

$$W = \begin{bmatrix} 1/8 & 1/8 & 1/4 & 0 & 1/2 & 0 & 0 & 0 \\ 1/8 & 1/8 & 1/4 & 0 & -1/2 & 0 & 0 & 0 \\ 1/8 & 1/8 & -1/4 & 0 & 0 & 1/2 & 0 & 0 \\ 1/8 & 1/8 & -1/4 & 0 & 0 & -1/2 & 0 & 0 \\ 1/8 & -1/8 & 0 & 1/4 & 0 & 0 & 1/2 & 0 \\ 1/8 & -1/8 & 0 & 1/4 & 0 & 0 & -1/2 & 0 \\ 1/8 & -1/8 & 0 & -1/4 & 0 & 0 & 0 & 1/2 \\ 1/8 & -1/8 & 0 & -1/4 & 0 & 0 & 0 & -1/2 \end{bmatrix}$$

In general, Q=IW. Where Q is row transformed matrix and I is the original image matrix. Haar wavelet transformation does this transformation to each column of the image matrix. This is done by multiplying I on the left by the transpose of W. The final equation is the row and column transformed matrix T of I.

$$T = ((IW)^{TW})^{T} = W^{T}IW$$

To return row and column transformed matrix to original matrix, we use the following equation

$$I = ((T^{T}W^{-1})TW^{-1} = (W^{-1})T(W^{-1})$$

This method is called the inverse transformation method and used in image reconstruction. [6]
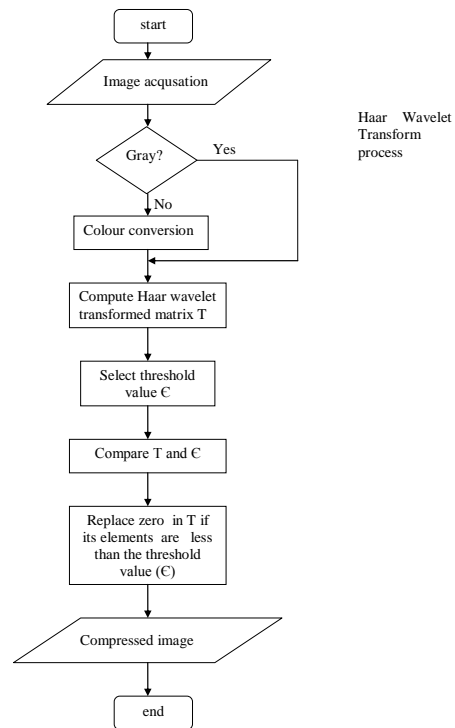
5. SYSTEM DESIGN

Figure 3. System Design of Image Compression

5.1. Steps in image compression

o Input image is read from some digital imaging devices and convert to matrix form.
o Compute the row-column transformed matrix (T) of original image matrix.
o Select the threshold value (Є) and compare with the transformed matrix.
o Replace zero in transformed matrix if the elements in the transformed matrix are less than or equal to threshold value (Є). Resulting matrix is known as compressed matrix.
o Compute the compression ratio using transformed matrix and compressed matrix.

6. COMPRESSION RATIO

The level of compression is measured by compression ratio. Compression is the ratio of nonzero elements in transformed matrix (T) to the number of nonzero elements in compressed matrix. If the selected threshold value is equal to zero no element of transformed matrix (T) is changed, and this compression is called lossless compression. If the selected threshold value is greater than zero this compression is called lossy compression.
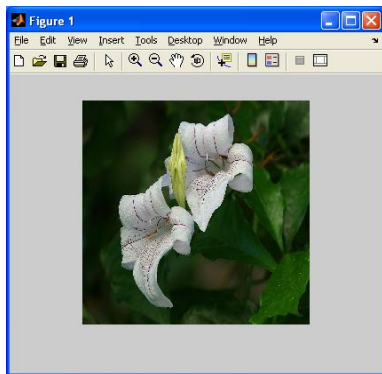
Figure 4. Original image of flower.jpg

7. IMPLEMENTATION

In the implementation of the system, input images are converted to matrix using MATLAB software and transformed by using Haar transformation matrix.
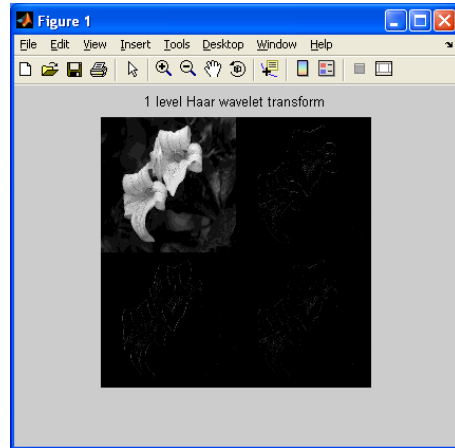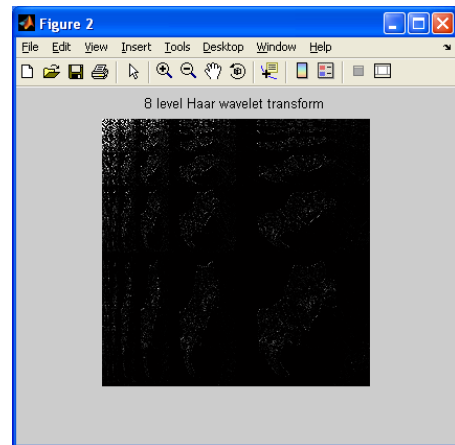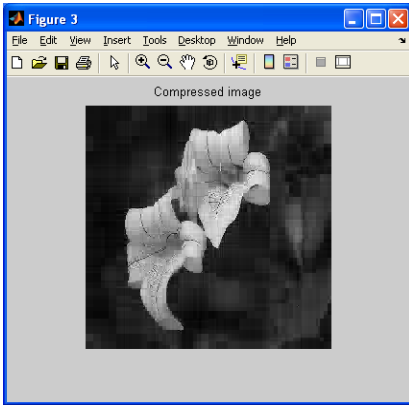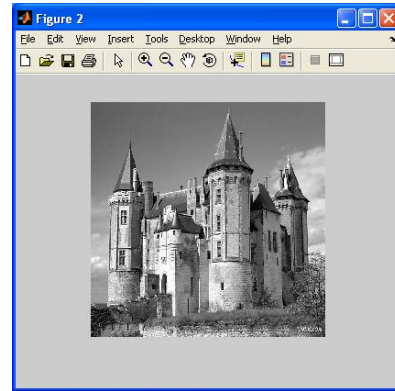

Figure 5. 1 level Haar wavelet transform

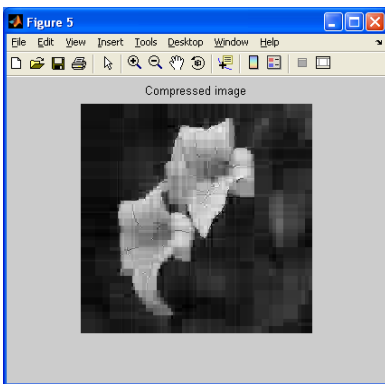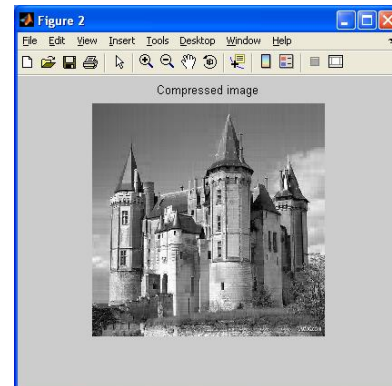**Figure 6**. 8 level Haar wavelet transform

Figure 7. Compressed image (Є =0)

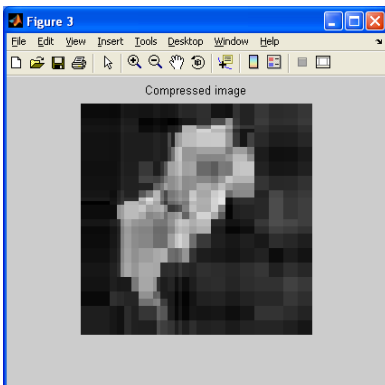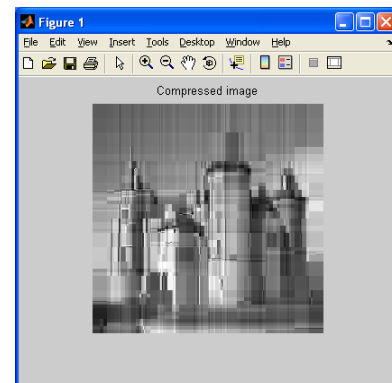**Figure 10.** Original image of castle.jpg

Figure 8. Compressed image (Є=1)

Figure 11. Compressed image (Є =0)

Figure 9. Compressed image (Є=6)

Figure 12. Compressed image (Є =5)

8. RESULT AND DISCUSSION

The result in Table 1 shows the compression ratio of test images.

Table 1. Experimental result of image compression

Test image	Compression ratio at Є=0
flower. jpg	24.7586 : 1
cameraman.tif	21.5366 : 1
Castle.jpg	5.3744 : 1
Asimo.jpg	6.5865 : 1
MRI.tif	28.0068 : 1
Autumn.tif	84.0205 : 1

9. CONCLUSION AND FUTHER WORK

This paper aims to improve image compression technique and to approach the Haar wavelet transform methods (averaging and differencing method and linear algebra method) in image compression. According to the experimental result, if the greater the threshold value (Є) then the larger the compression ratio. Compression ratio depends on the threshold value (Є). By using Haar wavelet transform method, it gets several advantages such as Haar wavelet transform is memory efficient, simplicity and the computation speed is high. This paper proposes by using two dimensional grayscale images. For limitation, this system cannot process color images. For further extension, color images can be compressed by other wavelet transform. Nowadays, image compression becomes common and crucial field. When images are compressed, we have so many advantages such as memory efficient and saving time.

REFERENCES

[1]   Greg Ames Dec 07, 2002. Image Compression,

[2]   Li Hong Huang Herman, Image Compression using Haar wavelet transform

[3]   Colm Mulcahy 1996, Image Compression Using Harr wavelet transform

[4]   Project One –Image compression http://www.owlnet.rice.edu/~elec539/Projects99/JAMK/projects1

[5]   Mike Oremus Ottopaskal Rice Dec 5 2000,Image Compression

[6]   Damien Adams and Halsey Patterson December 14,2006 " The Haar wavelet transform: Compression and Reconstruction''

[7]   www.wikipedia.org